

**DARK MATTER ANNIHILATION IN THE GRAVITATIONAL
FIELD OF A BLACK HOLE**

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In this paper we consider dark matter particle annihilation in the gravitational field of black holes. We obtain exact distribution function of the infalling dark matter particles, and compute the resulting flux and spectra of gamma rays coming from the objects. It is shown that the dark matter density significantly increases near a black hole. Particle collision energy becomes very high affecting relative cross-sections of various annihilation channels. We also discuss possible experimental consequences of these effects.

Keywords: Dark Matter; Black Hole; Gamma-Ray Astronomy.

1. Introduction.

The Dark Matter abundance of the Universe is now estimated as $\sim 23\%$ of the total energy content¹. The most popular hypothesis about its nature is that it consists of heavy stable elementary particles surviving from the early Universe. There are strong cosmological reasons to believe that the particles are very cold now ($v \ll c$).

In spite of no lack of theoretical candidates for the role of dark matter particle (hereafter DMP), its nature is still hazy. A very promising way to make the situation clearer is to observe astrophysical annihilation of the DMPs. In this article we consider dark matter annihilation in the gravitational field of a black hole. There are several reasons why this task is important. First of all, the greatest dark matter density in our Galaxy should occur in the center, and then the annihilation rate there must be the highest. A possible signal from the central source is a subject of wide speculation². Experimental data indicate a supermassive black hole in the centre of the Galaxy³. Secondly, the dark matter density should significantly increase near a compact massive object (see below). Thirdly, as it will be shown in this article, some very specific physical effects can appear nearby a black hole.

The influence of the black hole gravitational field on the dark matter annihilation has already been considered⁴, but the physical models were rather rough. In particular, they completely disregarded a significant increase of the DMP velocities

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and strong anisotropy of their phase space distribution. Meanwhile, as it will be shown below, the accreting DMP velocities are comparable with the speed of light. If we surmise the DMP mass to be equal 40 GeV, the particle collision energy can reach hundreds of GeVs. At such high energies the cross-sections of various annihilation channels vary significantly. This can strongly affect the annihilation rate and annihilation products.

The plan of our work is the following: first of all we obtain the exact phase distribution function for noninteracting particles accreting on a Schwarzschild (nonrotating) black hole in the spherically symmetric case. We assumed for simplicity that the particles far from the black hole are nonrelativistic ($v \ll c$). This assumption is well motivated in physically important cases, as it will be shown below. Then we calculate the number of collisions and the product distribution, and finally compute the product outlet to the distant observer. In conclusion, we discuss the obtained results.

2. Dark Matter Particle Phase Distribution in the Vicinity of a Black Hole

The gravitational field of a non-rotating black hole of mass M_{BH} in vacuum can be described by the Schwarzschild metric:

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{dr^2}{\left(1 - \frac{r_g}{r}\right)} - r^2 (\sin^2 \zeta d\zeta^2 + d\zeta^2) \quad (1)$$

where $r_g \equiv \frac{2GM_{BH}}{c^2}$ and c is the speed of light. The trajectory of a particle falling onto a Schwarzschild black hole can be written (Ref. 5, equations 101.4 and 101.5) as:

$$ct = \frac{E_0}{mc^2} \int \frac{dr}{\left(1 - \frac{r_g}{r}\right) \left[\left(\frac{E_0}{mc^2}\right)^2 - \left(1 + \frac{M^2}{m^2 c^2 r^2}\right) \left(1 - \frac{r_g}{r}\right) \right]^{1/2}} \quad (2)$$

$$\phi = \int \frac{M dr}{r^2 \left[\frac{E_0^2}{c^2} - \left(m^2 c^2 + \frac{M^2}{r^2}\right) \left(1 - \frac{r_g}{r}\right) \right]^{1/2}} \quad (3)$$

where E_0 , m and M are the total energy, mass and angular momentum of the particle respectively; E_0 and M are, of course, integrals of motion. As we have already mentioned, there are strong reasons to believe that the dark matter particles are cold. Then their speed is mainly determined in the Galaxy by its gravitational potential and can be estimated as a few hundred kilometers per second. Consequently, the considered particles are nonrelativistic far from the black hole which allows us to simplify the equations. Besides, we introduce here the system of units in which

the speed of light and the gravitational radius of the black hole are equal to the dimensionless unity: $c = 1$, $r_g = 1$. Then equations (2,3) can be rewritten as:

$$t = \int \frac{r^2 \sqrt{r} dr}{(r-1) \sqrt{r^2 - \alpha^2(r-1)}} \quad (4)$$

$$\phi = \int \frac{\alpha dr}{\sqrt{r} \sqrt{r^2 - \alpha^2(r-1)}} \quad (5)$$

where $\alpha \equiv \frac{M}{mc}$. For the radial component v_r , tangential component v_{tan} and the module v of the particle velocity^a we obtain:

$$v_r = \frac{\sqrt{r^2 - \alpha^2(r-1)}}{r\sqrt{r}}; \quad v_{tan} = \frac{\alpha}{r} \sqrt{\frac{r-1}{r}} \quad (6)$$

$$v = \frac{1}{\sqrt{r}} \quad (7)$$

Hence, the angle θ between the direction to the center of the black hole and the particle trajectory is:

$$\sin \theta = \frac{\alpha}{r} \sqrt{r-1}; \quad \cos \theta = \frac{\sqrt{r^2 - \alpha^2(r-1)}}{r} \quad (8)$$

From (7) one can see that all the particles reaching a radius r have the same velocity module. It is convenient to introduce the quantity \aleph of particles in a unit volume moving at the angle θ to the direction to the center of the black hole in a unit solid angle. Then the quantity of the particles crossing the sphere of the radius r in a unit time in a unit solid angle in the phase space is^b:

$$4\pi r^2 \aleph v \cos \theta d\tau d\Omega \quad (9)$$

If r is fixed, we obtain from (8):

$$\cos \theta d\Omega = \cos \theta d(2\pi(1 - \cos \theta)) = \pi d(\sin^2 \theta) = \pi d\left(\frac{\alpha^2(r-1)}{r^2}\right) = \pi \frac{r-1}{r^2} d\alpha^2 \quad (10)$$

Besides, we substitute the interval of physical time $d\tau$ for the interval of the universal Schwarzschild time coordinate $dt = d\tau \sqrt{1 - \frac{1}{r}}$. Then we can rewrite (9) as:

$$4\pi^2 \aleph \frac{(\sqrt{r-1})^3}{r} d\alpha^2 dt \quad (11)$$

Let us consider the particles from the interval $[\alpha; \alpha + d\alpha]$; α is an integral of motion, so α and $d\alpha$ remain constant along the bundle of trajectories. Since the system is stationary, the quantity of the particles considered crossing any sphere of the radius

^aBy v_r , v_{tan} , and v we imply the true, physical components of the velocity, in contrast to the coordinate ones. For example, v_r is equal to $\frac{\sqrt{-g_{rr}}}{\sqrt{g_{tt}}} \frac{dr}{dt}$ instead of $\frac{dr}{dt}$

^bWe use that in the Schwarzschild metrics the surface of a sphere of radius r is equal to $4\pi r^2$.

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r (if they cross it at all) in a unit of the Schwarzschild time t must be equal^c. From (11) we obtain:

$$4\pi^2 \aleph \frac{(\sqrt{r-1})^3}{r} = \text{const} \quad (12)$$

In order to find \aleph , we should impose a boundary condition on the infinity. From (12) it follows that $\aleph \rightarrow 0$ if $r \rightarrow \infty$, while it would be reasonable to expect that $\aleph \rightarrow n_\infty/4\pi$, where n_∞ is the dark matter particle concentration at infinity. The contradiction is a consequence of our assumption that the velocity of the particles at infinity v_∞ is zero. According to Ref. 5, the gravitational capture cross-section of nonrelativistic particles is:

$$\sigma = 4\pi r_g^2 \left(\frac{c}{v_\infty} \right)^2$$

The cross-section becomes infinite if $v_\infty \rightarrow 0$. In order to avoid this difficulty, we should consider that in practical situations v_∞ is never equal to 0 exactly: for example, near the Earth the dark matter particle speed is $v_\infty \sim 300 \text{ km/s}$. We cut the solution (12) on the radius r_∞ , where the velocity given by (7) becomes equal to v_∞ . According to (7), $r_\infty = \frac{1}{v_\infty^2}$. The dark matter distribution at larger distances will be considered undisturbed by the gravitational field of the black hole^d. Then the quantity of the particles crossing the sphere of the radius r_∞ in a unit time in a unit solid angle in the phase space is:

$$4\pi r_\infty^2 n_\infty v_\infty \cos \theta \frac{d\Omega}{4\pi} d\tau = r_\infty^2 n_\infty v_\infty \cos \theta d\Omega d\tau \quad (13)$$

According to (10) $\cos \theta d\Omega = \pi \frac{r-1}{r^2} d\alpha^2$. As $r_\infty \gg 1$, we obtain $\cos \theta d\Omega = \frac{\pi d\alpha^2}{r_\infty}$; $d\tau = dt$. Then we can rewrite (13) as:

$$\pi \frac{n_\infty}{v_\infty} d\alpha^2 dt \quad (14)$$

Comparing equations (12), (13) and (14), we derive that the constant in (12) is equal to:

$$\text{const} = \pi \frac{n_\infty}{v_\infty}$$

Substituting it into (12), we obtain:

$$4\pi^2 \aleph \frac{(\sqrt{r-1})^3}{r} = \pi \frac{n_\infty}{v_\infty} \quad (15)$$

$$\aleph = \frac{n_\infty}{4\pi v_\infty} \frac{r}{(\sqrt{r-1})^3}$$

^cWe use also that the Schwarzschild metrics is static.

^dThis supposition is true if $v_\infty \ll c$. In fact, if $v_\infty = 300 \text{ km/s}$, then $r_\infty = 10^6 r_g$ i.e. gravitational field on the radius r_∞ is weak

Deducing the initial formula (12) and considering the particles crossing the sphere r at the angle θ to the direction to the center, we imply that their world lines begin at infinity. It is true for the angles

$$\cos \theta \geq \frac{\sqrt{r^2 - 4(r-1)}}{r} \Xi(2-r) \quad (16)$$

where

$$\Xi(x) = \begin{cases} 1, & x > 0 \\ 0, & x = 0 \\ -1, & x < 0 \end{cases}$$

(see the details about particle trajectories in the Schwarzschild field in Ref. 5). The world lines, for which the condition (16) fails, should begin on the event horizon. Of course, there are no particles in these angles. Finally, we obtain for \aleph :

$$\aleph = \begin{cases} \frac{n_\infty}{4\pi v_\infty} \frac{r}{(\sqrt{r-1})^3}, & \cos \theta \geq \frac{\sqrt{r^2 - 4(r-1)}}{r} \Xi(2-r) \\ 0, & \cos \theta < \frac{\sqrt{r^2 - 4(r-1)}}{r} \Xi(2-r) \end{cases} \quad (17)$$

This is the formula which describes the collapsing particle distribution around the black hole.

3. Calculations and Results

Using the distribution function (17) of DMPs, one can calculate the annihilation signal (measuring by a distant observer) for various annihilation channels and various dependencies $\sigma(E)$. As an illustration, we consider the following important model process: a particle and an antiparticle of the dark matter annihilate into two photons ($\chi\bar{\chi} \rightarrow 2\gamma$). The DMP mass is assumed to be $m_{\text{DM}} = 40$ GeV. The cross-section is described by the Breit-Wigner formula:

$$\sigma(E) = \sigma_0 \frac{(\Gamma/2)^2}{(E - E_r)^2 + (\Gamma/2)^2} \quad (18)$$

Here E is energy in the c.m. system, E_r is the resonance energy. We take $E_r = 91$ GeV, $\Gamma = 2.5$ GeV which approximately corresponds to the values of Z_0 boson, though we do not propose any concrete annihilation process, considering just a model problem. We suppose $\sigma_0 = 10^{-4}$ GeV $^{-2}$ in order to provide a characteristic weak cross-section out of the resonance $\sigma(50\text{GeV}) \simeq 10^{-8}$ GeV $^{-2}$. The photons were assumed to outcome spherically symmetrically in the c.m. system. The photon energy, of course, increases with the collision energy. Besides, the centre of mass of the colliding particles is generally speaking not at rest. So in the laboratory reference system the photons are created asymmetrically, and their energy depends on the angle of producing.

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We use stationarity of the problem to calculate the photon outgoing to infinity. The general theory of relativity effects are taken into proper account. First, the photons undergo redshift: a photon produced on the radius r with the energy E_0 outcomes to a distant observer with energy

$$E = E_0 \sqrt{1 - \frac{r_g}{r}} \quad (19)$$

Second, the photons produced at the angles

$$\cos \theta \geq \sqrt{1 - \frac{27}{4} \frac{(r-1)}{r^3}} \Xi(r - \frac{3}{2}) \quad (20)$$

are captured by the black hole⁷.

We assume the black hole mass to be equal $M_{BH} = 3 \cdot 10^6 M_\odot$ (which corresponds to $r_g = 10^{12}$ cm), $v_\infty = 300$ km/s, and $n_\infty = 2 \cdot 10^{-2}$ cm⁻³ (~ 0.8 GeV/cm³). The last value is characteristic at the Sun system vicinity, for the Galaxy centre it must be underestimated. Unfortunately, there is no reliable estimation for the dark matter density in the centre of the Galaxy. The results of the calculations are represented in Fig. 1.

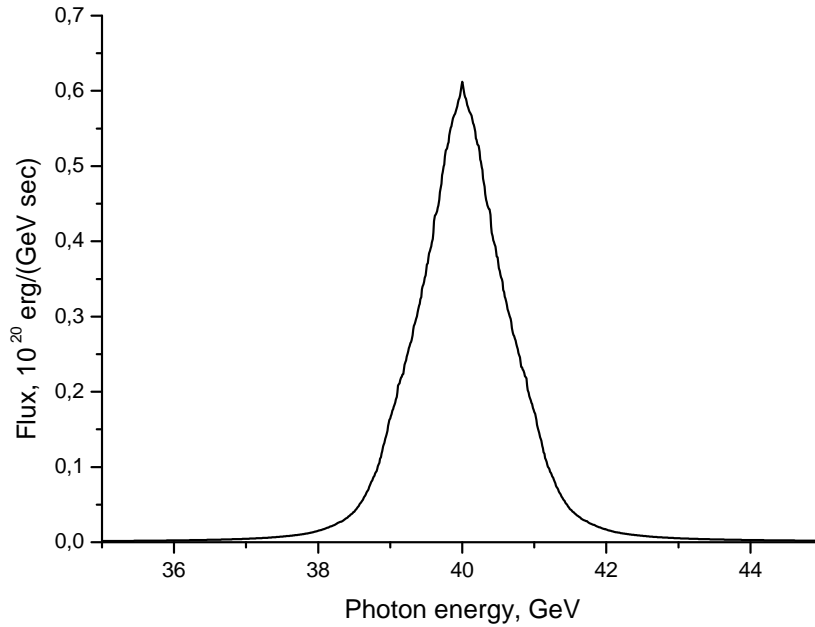


Fig. 1. The spectrum of gamma-ray emission produced by the dark matter annihilation into two photons. The cross-section is described by (18)

4. Discussion

Let us discuss the obtained results, giving special heed to the possible observable effects.

First of all, according to (17), particle concentration near the black hole ($r \sim r_g$) is proportional to n_∞/v_∞ (we should remind that n_∞ and v_∞ are concentration and velocity of DMPs undisturbed by the gravitational field). Since the annihilation signal intensity is proportional to the squared concentration, the signal grows by the factor of the order of $(c/v_\infty)^2$ with respect to the undisturbed dark matter field. As it has been already mentioned, the dark matter is cold; the speed of its thermal motion is negligible. The DMP velocity in the Galaxy is mainly determined by the gravitational potential of the sighting point. In the vicinity of the Sun system $v_\infty \simeq 300$ km/s. Near the Galaxy centre v_∞ is bigger but of the same order. Consequently, one can expect the signal augmentation by six orders only due to the dark matter concentration growth. Unfortunately, the effective region volume is not so large (of an order of πr_g^3), but for the supermassive central black hole in our Galaxy $r_g \simeq 10^7$ km.

Secondly, though the DMP speed near the black hole tends to the speed of light, the energy of their collisions does not excel a finite limit. Indeed, using (6,7) one can make sure that the sum energy of two colliding DMPs in the center-of-mass system is equal:

$$E = 2m \frac{\sqrt{1 - v^2 \cos^2 \beta}}{\sqrt{1 - v^2}} = 2m \frac{\sqrt{r - \cos^2 \beta}}{\sqrt{r - 1}} \quad (21)$$

where 2β is the angle between the velocities of the particles. The last radius, where the head-on collisions (with $\beta = \pi/2$) are still possible, is $r = 2$. The corresponding collision energy is $2\sqrt{2}m$. When particles get closer to the horizon, their speeds remain to grow, but possible β rapidly decreases (since the velocities of all the falling particles become almost parallel). When $r < 2$, it is determined by (16). Substituting (16) to (21), we obtain:

$$E_{max} = 2m\sqrt{r^2 + 4}$$

$$\lim_{r \rightarrow 1} E_{max} = 2\sqrt{5} m \quad (22)$$

Thus, the maximal collision energy nowhere surpasses $2\sqrt{5} m$. Practical conclusion from this fact is that if the threshold energy of some DMP interaction process excels $2\sqrt{5} m$, the process is impossible. Nevertheless, for 40 GeV particles the limit is ~ 180 GeV.

So the energy of the DMP collisions can be very high, which can significantly affect the cross-sections of various annihilation channels and even change the main one. In particular, strong resonances at the energies around Z_0 and Higgs bosons masses can appear. Of course, it could change the annihilation products and strongly influence the annihilation rate.

Thirdly, let us consider the resulting spectrum of the annihilation signal. Under usual situations (far from compact objects) the considered way of annihilation ($\chi\tilde{\chi} \rightarrow 2\gamma$) should produce a very narrow line: the width of photon spectral distribution is determined by the particle energy spread:

$$\Delta E \sim m_{\text{DM}} v^2 \quad (23)$$

In the considered case there are at least three mechanisms of the line broadening and shifting. First, the annihilating particle collision energy significantly depends on r and the angle between particle velocities (see equation 21). The energy of produced photons varies correspondingly. Second, the centre of mass of the colliding particles is generally speaking not at rest. Consequently, Lorentz shifting appears. Third, gravitational redshift quite strongly decreases the energy of the photons generated near r_g . We should mention that the effects of the first and third mechanisms are opposite in sign: so they can compensate each other. Nevertheless, one could expect that the annihilation line was arbitrarily wide.

In Fig. 1 we can see that various broadening effects well compensate each other, and the line remains respectively narrow and unshifted. This fact is of big practical significance. Indeed, different ways of dark matter annihilation signal detection from the central Galactic source (where a supermassive black hole is situated, as we have already mentioned) are discussed in the literature². It is suggested to observe immediate annihilation products as well as the synchrotron emission radiated by them. However, a serious problem appears here. The concentration of not only dark, but of normal matter too is high in the centre of the Galaxy. The normal matter accretion on the black hole is accompanied by complex magnetohydrodynamic effects which can generate high-energy particles, synchrotron emission etc. of various spectra and characteristics. Since the cross-section of interaction of baryonic matter is incommensurably greater, effectiveness of these processes is very big. Therefore, even if some interesting signal from the Galactic centre has been observed, one should prove that it is the dark matter annihilation that gives it. The case when the emission spectrum is wide it's a very difficult task. Whatever is the signal, to fit it by the ordinary matter emission is usually possible. In the case of a narrow line the situation is much better. It is more complicated to offer an alternative explanation to a narrow line on the energy > 10 GeV. Moreover, the energy of the line would immediately give us the DMP mass. Therefore, the study of the narrow annihilation line generation opportunity is a very promising problem, even if the relative cross-section of the channel is small. As we have seen, the DMP density near a black hole strongly increases. Besides, the collision energy growth can strongly elevate the two-photon annihilation channel efficiency. Thus, we can expect an intensive line formation about a black hole. A misgiving appears, however, that the line can be transformed into a wide band ($\Delta E/E \sim 1$) by the gravitational redshift, for instance. Figure 1 shows that it does not happen. The line remains narrow enough ($\Delta E/E \sim 0.05$). On the other hand, the line turns out to be distinctly wider then if it appears far from compact objects. In the last case, the width can be estimated via

(23) which gives us $\Delta E/E \sim (v_\infty/c)^2 \simeq 10^{-6}$. So the line generated near a black hole can be easily distinguished from the line appearing under usual situations.

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